Fen-Edebiyat Fakültesi	MIDETERM EXAM II	
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Signature:	Exam Date: 11/12/2018	

Each problem is worth 25 points. Duration is 80 minutes.

1. Find the general solution of $ty'' - (t+1)y' + y = t^2$, given that $y_1(t) = e^t$ and $y_2(t) = t+1$ are solutions of $ty'' - (t+1)y' + y = 0$. İpucu: $\int e^{-t}(t+a)dt = -e^{-t}(t+a+1) + C$.

Solution: $y = u_1(t)y_1(t) + u_2(t)y_2(t)$. Write the equation in the form $y'' - \frac{(t+1)}{t}y' + \frac{1}{t}y = t$. Then

$$u_1 = \int \frac{-y_2 g(t)}{W(t)}dt, \qquad u_2 = u_1 = \int \frac{y_1 g(t)}{W(t)}dt, \qquad g(t) = t, \qquad W(t) = y_1y_2' - y_2y_1' = -e^t t$$

We get

$$u_1(t) = \int e^{-t}(t+1)dt = e^{-t}(t+2) + c_1, \qquad u_2 = -t + c_2$$

The general solution is:

$$y(t) = (-e^{-t}(t+2) + c_1)e^t + (-t + c_2)(t+1) = c_1e^t + c_2(t+1) - t^2 - 2t - 2.$$

2. Find the form of the particular solution of the equation $y''' + 2y'' + y' = 4te^{-t} + \sin(3t)e^{2t} + 8$. (Do not compute the coefficients $(A, B, ...)$ of the particular solution)

Solution: Characteristic equation is $r(r+1)^2 = 0$ so the solution of the homogeneous equation is $y_h(t) = c_1 + c_2e^{-t} + c_3te^{-t}$. The form of the particular solution is
$Y_p = t^2(A+Bt)e^{-t} + (C\cos 3t + D\sin 3t)e^{2t} + Et$.

3. Use the method of undetermined coefficients to solve $y'' + y = e^t$, $y(0) = 0$, $y'(0) = 1$

Solution: The characteristic equation is $r^2+1 = 0$, $r = \pm i$. The homogeneous solution is $y_h = c_1\cos t + c_2\sin t$. Particular solution is $Y_p = Ae^t$. Plugging into equation, $2Ae^t = e^t$, $A = 1/2$. The solution is $y = c_1\cos t + c_2\sin t + \frac{1}{2}e^t$. Using initial conditions, we get the answer: $y(t) = \frac{1}{2}\cos t - \frac{1}{2}\sin t + \frac{1}{2}e^t$.

4. Find the power series solution in powers of x of the equation $y'' - y' = 0$, $y(0) = 1$, $y'(0) = 2$ up to and including fourth power of x.

Solution: Answer: $y(t) = a_0 + a_1x + \frac{a_1}{2!}x^2 + \frac{a_1}{3!}x^3 + \frac{a_1}{4!}x^4 + \cdots$. Use the initial conditions to find $a_0 = 1$, $a_1 = 2$. Find y', y'', plug into equation to get
$y(t) = 1 + 2x + x^2 + \frac{1}{3}x^3 + \frac{1}{12}x^4 + \cdots$